MATHEMATICAL SOLUTIONS FOR OPTIMAL DIMENSIONING OF NUMBER AND HEIGHTS OF SOME HYDROTECHNIQUE ON TORRENTIAL FORMATION

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Abstract
This paper is intended to achieve a mathematical model for the optimal dimensioning of number and heights of some dams/thresholds during a downpour, a decrease of water flow rate being obtained and by the solid material depositions behind the constructions a new smaller slope of the valley that changes the torrential nature that had before the construction is obtained.
The choice of the dam and its characteristic dimensions may be an optimization issue and the location of dams on the torrential (rainfall) aspect is dictated by the capabilities of the foundation and restraint so that the chosen solutions will have to comply with these sites. Finally, the choice of optimal solution to limit torrential (rainfall) aspect will be based on a calculation where the number of thresholds / dams can be a variable related to this, their height properly varying.
The calculation method presented is an attempt to demonstrate the multiple opportunities available to implement a technical issue solving conditions offered by the mathematics against soil erosion, which now is currently very topical on the environmental protection.

Keywords: torrential (rainfall) aspect, dams/ thresholds, optimization.

1. INTRODUCTION

In hillocky and hilly area, the rains are torrential, which even at an average intensity, produce the high flow capacities of water with massive transport of solid material.
On torrential formations from these area the cross works such as earth thresholds and dams, stone or concrete thresholds, or mixed are made.
Objectives:
- prevent soil erosion and avoiding floods;
- optimal choice of type of dam leads to a lower-cost investment.

It often finds it is necessary to make a check of general solution to be applied to a torrential (rainfall) formation on the dams type, their number and their heights of retention and to meet the general relation applicable to investment activities, ie [3]:

$$FO = \sum_{i=1}^{M}\sum_{j=1}^{N}\left(I_{ij} + T, Ch_{ij}\right) = MIN \qquad (1)$$

where:
I_{ij} = investment in the dam *i* of section *j* with N sections ;
T = The normalized time of recovery (years)
Ch_{ij} = annual expenses and repair the dam *i* of section *j*.
The objective function described above is sufficiently general.
It was noted that the optimization that is usually made by a designer, refers essentially to two dimensions balances of the dam designed, one on the horizontal, the cross profile of the valley and another on the vertical of the same profile [2].
The formation was divided into three sections conventionally chosen; the lengths and heights of these sections were projected on the horizontal axis and on the vertical one (Fig. 1).
Calculations made aimed at the achieving compliance with the adopted scale, the results appear in the horizontal and vertical projections, lengths and heights of sections, respectively.
From this figure also clear that there is a way of transforming a projection size from one to another, which is obtained by multiplying the tangent of the angle formed.
These possibilities are used for the linear programming optimization method, where the dimensions balances are made.

2. MATERIAL AND METHOD

The theoretical base is the solution of two surveys designed horizontal and vertical dimensions [1]:
a) horizontal

$$\sum_{i=1}^{M}\sum_{j=1}^{N} L_{ij} \leq LV \qquad (2)$$

where:
L_{ij} = horizontal projection of an element *i* existing or designed on a section *j* (not necessarily assume that L_{ij} is equal to the length of the section j).
L= valley length with j sections.
b) vertical

$$\sum_{i=1}^{M}\sum_{j=1}^{N} H_{ij} \geq HV \qquad (3)$$

where:
H = vertical projection of height of an element *i* existing or projected on the line *j*
HV = total height of torrential valley with *j* sections.

Obviously that H_{ij} can be ≶ Hij than the height of a section *HTj*.
Where appropriate, these considerations may be supplemented with other relationships that must be met, such as, for example, limiting the height of the dams, etc.
Relationship and process of transformation from the horizontal into the vertical one and inversely is known as:

$$L_{ij} = H_{ij} \operatorname{cotg} P_i \qquad (4)$$

where P_i = is an angle that makes the element *z* with horizontal, for example, the slope *I* of land (Fig. 1) or the slope *P* of alluvia deposit carried by waters P2.
For ease of exposure, in Figure 1 the matrix of an issue that was solved as an example title has been restored; for different applications the matrix model shown can be used; but one more developed may be used, too.
From this matrix a series of restrictions taken into account by the authors and are not limiting can be seen.
The entire length of torrential formation is divided into sections which are characterized by a length, average slope and eventually the liquid flow and solid, respectively
Within each section can be assumed to build more dams; in this case to simplify the example only two dams and several types has been considered. As part of each dam, three types of variables dam have been indicated, as follows:
Type 1: S and R - that represent the reserve spaces before the dam (S) and at the end of alluvia deposit carried by waters (R); in some cases these spaces may be cancelled.
The 2nd type of variable is Aij and represents the length of alluvia deposit carried by waters of elements i of the section j (Fig. 1) in a horizontal plane; the vertical projection it is as:

$$A_{ij} = H_{ij}/I_j \qquad (5)$$

where:
H_{ij} - the length of alluvia deposit carried by waters destined the element (to dam) i of section j; m;
I_j – the land slope in the section j; m/m;
The 3rd type of variable is P = the increase of slope at alluvia deposit carried by waters and performs a longitudinal slope of the order of 1-2% (Fig. 2).
In preparing the matrix the possibility that the latest dam in a section j can be extended with the alluvia deposit carried by waters also in the next section (*j* +1) and even in the post next (*j* +2) has been also considered.
This option allows, for a valley with a large number of sections, capturing as more accurate as possible an optimal solution.
Worthy of consideration is that in the taken example the proposed objective function is not economic, but a functional one functional namely, to minimize the free spaces S and H between the dams has been requested [2]:

$$FO = \sum_{i=1}^{M}\sum_{j=1}^{N}\left(S_{ij} + R_{ij}\right) = MIN \qquad (6)$$

where:
S_{ij} si R_{ij} = free spaces downstream of the dam and upstream of the dam alluvia deposit carried by waters of dam i of section *j*.
If it would have used equation (1) then it be taken into account indices of investment (lei/m3) and maintenance costs (lei/km and 10 years) variables with type of dam.
Restrictions of the model are (in order from the matrix):
- *The first group of limitations* refers to the condition that the horizontal projections of alluvia deposit carried by waters of dam,

including the increase of slope as the safety ranges to be exactly framed in the length of that section.

$$\sum_{k=1}^{k=L_1} L_{K_1 j} = LT_j \qquad (7)$$

LT_j = length of secrtion j (m);
k = number of components
k = 1: space in front of dam (m);
k = 2: length of alluvia deposit carried by waters
k = 3: increase of high (m);
k = 4: reserve space (m);
Reserve spaces *S* and *R* play a compensatory role, for example, if the solution does not exist – for making of dam – in the respective section.
- *The second restriction* refers to the limiting of dams high at a dimension possibly considered for the current technology. Geometrically, a dam height is proportional to the length of alluvia deposit carried by waters;

$$H_{ij} = A_{ij} \,.\, I_j \qquad (8)$$

If the dam height H_{ij} exceeds a height HT_j (of section j) then:

$$\sum_{i=1}^{M} A_{ij} I_j < HT_j \,, \quad (j = 1,2, \dots N) \qquad (9)$$

These conditions are very important because the actual local conditions of execution of a work limiting it strictly to the extant possibilities come into calculation.
Combined with the possibility to establish also location where it can be made a dam - by imposing some values to variables *S* and *H* described above, the sought optimal solution, to be a real part on the one hand, but on the other hand reduces the range of adopted possible solutions.
- *The third restriction* refers to the vertical closure, so the sum of components (dam height, the slope increase height of the alluvia deposit carried by waters surface,
etc.) existing in a section to be equal to the height of section *HTj*, as follows:

$$I_j \sum_{k=1}^{4} L_{k,j} = HT_j \,, \quad j = 1,2 \dots N \qquad (10)$$

$L_{k,j}$ – length of element designed.
In case that, for some reason, the dam height should be limited - constructively (does not allow execution material) or geotechnically (foundation), etc., then this restriction is written properly to situation and instead HTj H_{real} or bound by these conditions will be considered.
- *A fourth restriction* refers to meet the accumulation role of alluvia deposit carried by waters behind the dam. The restriction is written on both sections, because of different torrential character of diverse parts of the valley and whole torrential valley.

$$I_{ij} \sum_{k=1}^{2} V_h \geq VT_{ij} \,, \qquad (11)$$

V_h = voloume of alluvia deposit carried by waters as indicator relative to the length alluvia deposit (m^3/m);
VT_{ij} = total volume of alluvia deposits carried by waters bound to accumulate along section j (m^3);
h = index of dams number in a section (in taken example h = 2);
This restriction uses an index (Vh) adapted to the rigorous of calculation.
The alluvia deposit carried by waters volume is usually inferred from the length or height of the dam squared;
Since currently the programming programs with variables to square are difficult for large models, this way to avoid variables squared has been adopted.
Using an average index (Vh), as it was done above, indeed brings, a local relativization into calculation, but that subsequently it can be undone: from the experience of several runs it has come out that this source of errors is negligible compared with other errors that exist in the procedures for determining the optimal solution by other means of development.
- *A fifth restriction* refers to the limitation of consumption of building materials, related to the height of the dam.

$$M_q \sum_{j=1}^{N} A_{ij} I_j \leq MT_q^l \,, \; (q = 1,2, \dots, R) \qquad (12)$$

where:
M_q^i = consumption specification *q* (m^3/m);
MT_q = consumption index of material q (m^3).
- *Restriction 6* has a more special character and refers to the proportion that should exist between the alluvia deposit carried by waters length and the slope increase of surface of this.

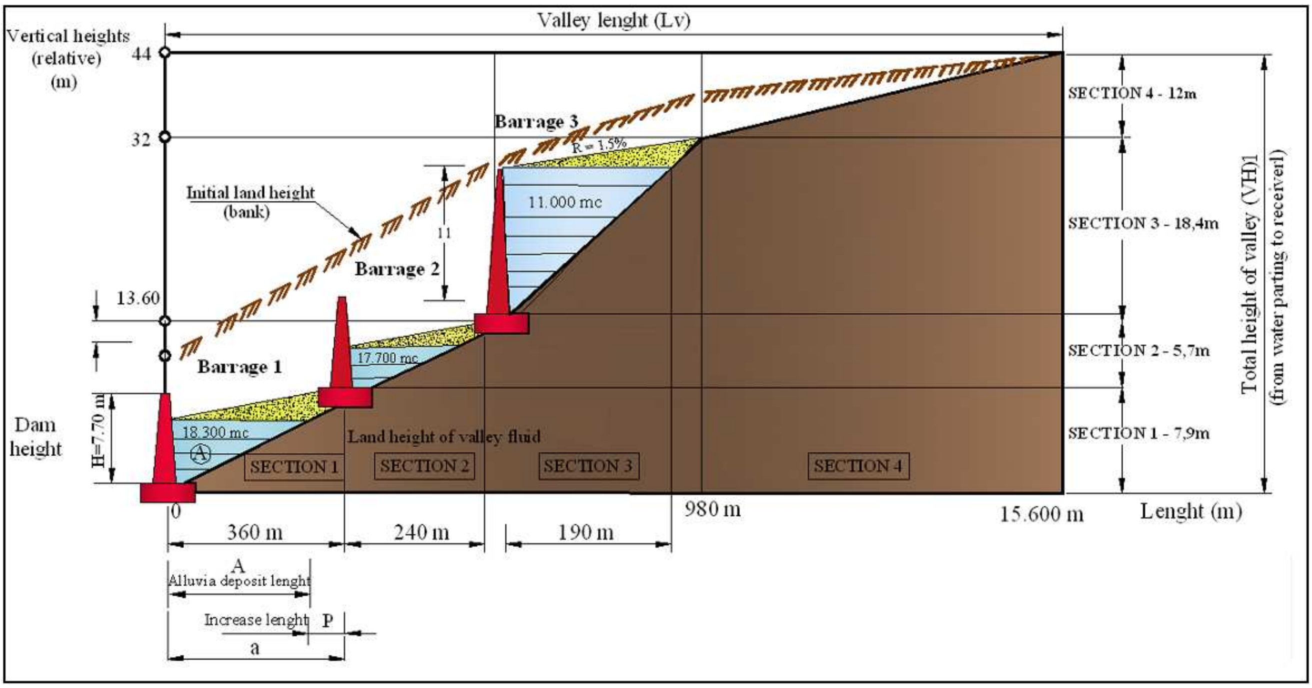

Figure 1 Cross profile through the torrential valley (rainfall) and calculation elements of optimal solution

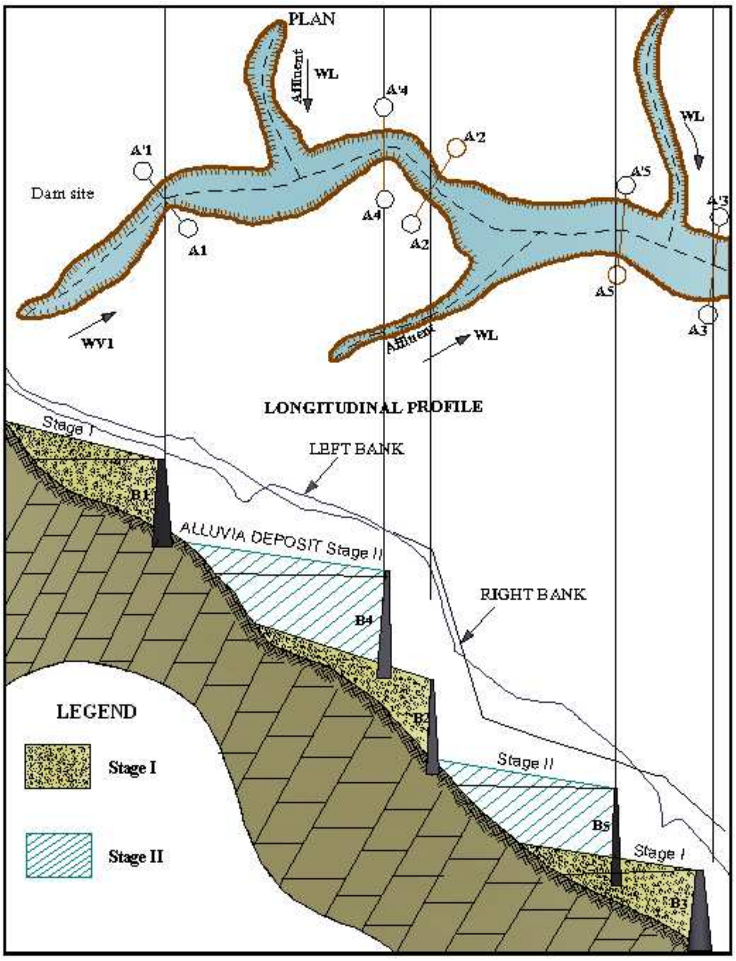

Figure 2 Torrential formation (rainfall) with location transverse location of works staged

3. RESULTS AND DISCUSSION

The results of such an application are shown in figure 2 and graphically in figure 1. Under the technical aspect, the optimal solution has been predictable because of the small sizes of the issue; in case of high torrential valleys (rainfalls), with many dams, the optimum solution arises much different from the one shown by the intuitive methods traditionally used.

4. CONCLUSION

Mathematical method used can be currently employed of design and will be generally succeed by the following stages;

1) analysis of longitudinal profile of the torrential valley (rainfall) and its segmentation into sections conventionally chosen;
2) projection lengths and heights of these sections on the horizontal axis, the vertical one, respectively and the mathematical solution of them;
3) calculation system variables are the area length of alluvia deposit carried by waters and length of slope increase;
4) choosing the optimal number of dams as well as correct dimensioning of them.

The calculation model presented is also an attempt to demonstrate the multiple opportunities available to implement a technical issue as solving given by the mathematical methods for the soil erosion control work.

Schematic forms used to simplify some aspects, as well as taking into account of a small parameters as against the real one are due to start phase in the field utilization mathematical methods in the work of CES, that will be upgraded to be more realistic in the design practice.

The optimum location of dams/thresholds in a torrential (rainfall) formation, as well as correct dimensioning of them has first of all an important impact on the hydraulic flow regime transformation, in order to reduce the hydraulic gradients/slope and hence to reduce the critical erosion velocity.

Also, secondly has a significant impact economically, in the sense that they can realize low-cost investment.

5. REFERENCES

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